Magnetic-field-induced reentrance of Fermi-liquid behavior and spin-lattice relaxation rates in $\mathrm{YbCu_{5-x}Au_x}$

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A strong departure from Landau-Fermi liquid (LFL) behavior have been recently revealed in observed anomalies in both the magnetic susceptibility χ and the muon and ^{63}Cu nuclear spin-lattice relaxation rates $1/T_1$ of $\mathrm{YbCu_{5-x}Au_x}$ ($x = 0.6$). We show that the above anomalies along with magnetic-field-induced reentrance of LFL properties are indeed determined by the scaling behavior of the quasiparticle effective mass. We obtain the scaling behavior theoretically utilizing our approach based on fermion condensation quantum phase transition (FCQPT) notion. Our theoretical analysis of experimental data on the base of FCQPT approach permits not only to explain above two experimental facts in a unified manner, but to clarify the physical reasons for a scaling behavior of the longitudinal magnetoresistance in $\mathrm{YbRh_2Si_2}$.

PACS numbers: 71.27.+a, 76.60.Es, 73.43.Qt

Landau Fermi liquid (LFL) theory designed to describe the thermodynamic, transport and relaxation properties of itinerant electron systems is perhaps the most fruitful theory in condensed matter physics [1]. The discovery of strongly correlated states characterized by the non-Fermi liquid (NFL) behavior of condensed matter in past century is still opening up new vistas in physics, projecting one of the tremendous challenges in modern condensed matter physics [2, 3, 4]. This behavior is so unusual that the traditional Landau quasiparticles paradigm does not apply to it. The paradigm states that the properties is determined by quasiparticles whose dispersion is characterized by the effective mass M^* which is independent of temperature T, density x, magnetic field B and other external parameters.

The experimental results collected on HF metals and 2D ^{3}He demonstrate the existence of very high values of a quasiparticle effective mass M^* or even its divergence [3, 4]. Earlier [5, 6], a concept of fermion condensation quantum phase transition (FCQPT) preserving quasiparticles and intimately related to the unlimited growth of M^*, had been suggested. Further studies [7, 8, 9] show that it is capable to deliver an adequate theoretical explanation of vast majority of experimental results in different strongly correlated Fermi-systems. In FCQPT approach, M^* starts to depend on T, x, B and other external parameters. However, the extended Landau quasiparticles paradigm is preserved. The main point here (see, e.g., [9] and references therein) is that, as before, the quasiparticles determine the physical properties of strongly correlated Fermi-systems while their effective mass is a function of external parameters. The FCQPT approach had been already successfully applied to describe the thermodynamic properties of such different strongly correlated systems as ^{3}He on one side and complicated heavy-fermion (HF) compounds on the other side [10, 11, 12].

One of the most interesting and puzzling issues in the research of HF metals is their anomalous dynamic and relaxation properties. It is important to verify whether quasiparticles with effective mass M^* still exist and determine the physical properties of muon and ^{63}Cu nuclear spin-lattice relaxation rates $1/T_1T$ in HF metals throughout their temperature - magnetic field phase diagram, see Fig.1. This phase diagram comprises both LFL and NFL regions as well as NFL-LFL transition one (below we call it crossover region), where magnetic-field-induced LFL reentrance occurs. Measurements of the muon and ^{63}Cu nuclear spin-lattice relaxation rates $1/T_1$ in $\mathrm{YbCu_{4.4}Au_{0.6}}$ [13] have shown that it differs substantially from ordinary Fermi liquids obeying Korringa law. Namely, it was reported that for $T \to 0$ reciprocal relaxation time diverges as $1/T_1T \propto T^{-4/3}$ following the behavior predicted by the self-consistent renormalization (SCR) theory. The static uniform susceptibility χ diverges as $\chi \propto T^{-2/3}$ so that $1/T_1T$ scales with χ^2. Latter result is at variance with SCR theory [13]. Moreover, the application of magnetic field B restores LFL behavior from initial NFL one, significantly reducing $1/T_1$ [13]. These experimental findings are hard to explain within both conventional LFL approach and in terms of other approaches like SCR theory [13, 14].

In this paper, we analyze $1/T_1T$ of $\mathrm{YbCu_{4.4}Au_{0.6}}$ and show that the observed data can be well captured utilizing the above FCQPT concept based on the extended quasiparticles paradigm. We demonstrate that the crossover is regulated by the universal behavior of the effective mass $M^*(B,T)$ observed in many HF metals. It is exhibited by $M^*(B,T)$ when HF metal transits from LFL regime (induced by a magnetic field application) to NFL one taking place at rising temperatures. We show that violations of the Korringa law come from the

dependence of M^* on magnetic field and temperature. Our calculations of $1/T_1T$ are in good agreement with experimental findings.

To discuss the deviations from Korringa law in light of NFL properties of $YbCu_{4.4}Au_{0.6}$, we notice that in LFL theory spin-lattice relaxation rate $1/T_1$ is determined by the quasiparticles near Fermi level. The above relaxation rate is related to the decay amplitude of the quasiparticles, which in turn is proportional to the density of states at the Fermi level $N(E_F)$. Formally, spin-lattice relaxation rate is determined by the imaginary part χ'' of the low-frequency dynamical magnetic susceptibility $\chi(\mathbf{q},\omega\to 0)$, averaged over momentum $\mathbf{q}$ [14]

$$\frac{1}{T_1}=\frac{3T}{4\mu_B^2}\sum_{\mathbf{q}}A_{\mathbf{q}}A_{-\mathbf{q}}\frac{\chi''(\mathbf{q},\omega)}{\omega}, \tag{1}$$

where $A_{\mathbf{q}}$ is the hyperfine coupling constant of the muon (or nuclei) with the spin excitations at wave vector $\mathbf{q}$, μ_B is Bohr magneton. If $A_{\mathbf{q}}\equiv A$ is independent of q, then standard LFL theory relation yields

$$\frac{1}{T_1T}=\pi A^2N^2(E_F), \tag{2}$$

Equation (2) can be viewed as Korringa law. Since in our FCQPT approach the physical properties of the system under consideration are determined by the effective mass $M^*(T,B,x)$, we express $1/T_1T$ in Eq. (2) via it. This is accomplished with the standard expression [1] $N(E_F)=M^*p_F/\pi^2$, rendering Eq. (2) to the form

$$\frac{1}{T_1T}=\frac{A^2p_F^2}{\pi^3}M^{*2}\equiv\eta\left[M^*(T,B,x)\right]^2, \tag{3}$$

where $\eta=(A^2p_F^2)/\pi^3$ =const. The experimentally observed relation in $YbCu_{5-x}Au_x$ [13]

$$\frac{1}{T_1T}\propto\chi^2(T) \tag{4}$$

follows explicitly from Eq. (3) and well-known LFL relations $M^*\propto\chi\propto C/T$.

Having derived explicit relation between $1/T_1T$ and quasiparticle effective mass, we are going to analyze the properties of latter. For that, we use the model of homogeneous HF liquid with the effective mass $M^*(T,B,x)$, where $x=p_F^3/3\pi^2$ is a number density and p_F is Fermi momentum [1]. This homogeneity permits to avoid complications associated with the crystalline anisotropy of solids [9]. We begin with the case when at $T\to 0$ the heavy-electron liquid behaves as LFL and is brought to the LFL side of FCQPT by tuning of a control parameter like x. At elevated temperatures the system transits to the NFL state. The dependence $M^*(T,x)$ is governed by Landau equation [1]

$$\frac{1}{M^*(T,x)}=\frac{1}{M}+\int\frac{\mathbf{p}_F\mathbf{p}}{p_F^3}F(\mathbf{p_F},\mathbf{p})\frac{\partial n(\mathbf{p},T,x)}{\partial p}\frac{d\mathbf{p}}{(2\pi)^3}, \tag{5}$$

where $n(\mathbf{p},T,x)$ is Fermi function, $F(\mathbf{p}_F,\mathbf{p})$ is Landau interaction amplitude and M is a free electron mass. At $T=0$, eq. (5) reads $M^*/M=1/(1-N_0F^1(p_F,p_F)/3)$ [1]. Here N_0 is the density of states of a free electron gas, $F^1(p_F,p_F)\equiv F^1(x)$ is the p-wave component of Landau interaction amplitude F. When at some critical point $x=x_{FC}$, $F^1(x)$ achieves certain threshold value, the denominator tends to zero and the system undergoes FCQPT related to divergency of the effective mass $M^*(x)/M=A+B/(x_{FC}-x)$, where A and B are parameters.

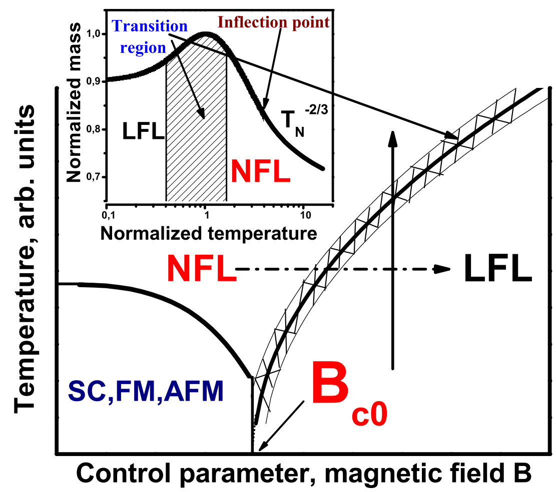

FIG. 1: Schematic phase diagram of HF metal. B_{c0} denotes the magnetic field at which the effective mass divergences. The vertical arrow shows the LFL-NFL transition at fixed B with M^* depending on temperature. The dash-dot horizontal arrow illustrates the system moving in the NFL-LFL direction along B at fixed temperature. At $B<B_{c0}$ the system can be in a superconducting (SC), ferromagnetic (FM) or antiferromagnetic (AFM) states. Inset reports a schematic plot of the normalized effective mass M_N^* versus the normalized temperature. Transition regime, where M_N^* reaches its maximum, is shown by the hatched area both in the inset and in the main panel. The inflection point in M_N^* is shown by the arrow.

A qualitative consideration of Eq. (5) [8, 10] shows that at lowest temperatures we have the LFL regime, see the inset to Fig. 1. Then the system enters the transition regime: M^* grows, reaching its maximum M_M^* at $T=T_M$, with subsequent diminishing. Near temperatures $T\geq T_M$ the last "traces" of LFL regime disappear and the NFL state takes place, manifesting itself in decreasing of M^* as $T^{-2/3}$ [10]. When the system is near FCQPT, it turns out that $M^*(T,x)$ can be well approximated by a simple universal interpolating function [9, 10]. The interpolation occurs between the LFL ($M^*\propto T^2$) and NFL ($M^*\propto T^{-2/3}$) regimes thus describing the above crossover [8, 10]. Introducing the dimensionless variable $y=T_N=T/T_M$, we obtain the desired expression

$$\frac{M^*(T/T_M)}{M_M^*}=M_N^*(y)\approx c_3\frac{1+c_1y^2}{1+c_2y^{8/3}}. \tag{6}$$

Here $M^*_N(y)$ is the normalized effective mass, c_1 and c_2 are parameters, obtained from the condition of best fit to experiment, c_3 ensures the normalization: $M_N(1) = 1$. As it follows from Eq. (6), M^* reaches the maximum M^*_M at some temperature T_M. Since there is no external physical scales near FCQPT point, the normalization of both M^* and T by internal parameters M^*_M and T_M immediately reveals the scaling behavior of the effective mass. The decay law $M^*_N \propto T_N^{-2/3}$ along with expression (3) permits to express the relaxation rate in this temperature range as

$$\frac{1}{T_1 T} = a_1 + a_2 T^{-4/3} \propto \chi^2(T), \tag{7}$$

where a_1 and a_2 are fitting parameters.

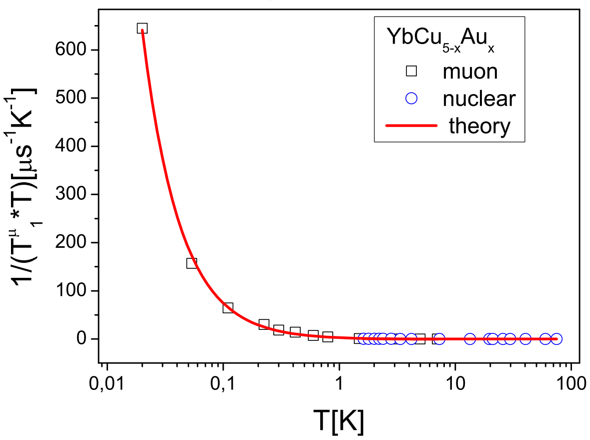

FIG. 2: Temperature dependence of muon (squares) and nuclear (circles) spin-lattice relaxation rates (divided by temperature) for $YbCu_{4.4}Au_{0.6}$ [13]. The solid curve is our theoretical expression (7).

The dependence (7) is reported in Fig. 2 along with experimental points for muon and nuclear spin-lattice relaxation rates in $YbCu_{4.4}Au_{0.6}$ [13]. It is seen from Fig. 2 that Eq. (7) gives pretty good description of the experiment in the extremely wide temperature range. This means that the extended Landau paradigm is valid and quasiparticles survive in close vicinity of FCQPT, while the observed violation of Korringa law comes from the dependence of the effective mass on temperature.

At small magnetic fields B (that means that Zeeman splitting is small), the effective mass does not depend on spin variable and B enters Eq. (5) as $B\mu_B/T$ making $T_M \propto B\mu_B$ [9, 10]. The application of magnetic field restores the LFL behavior, and at $T \leq T_M$ the effective mass depends on B as [9, 10]

$$M^*(B) \propto (B - B_{c0})^{-2/3}. \tag{8}$$

Note that in some cases $B_{c0} = 0$. In our simple model B_{c0} is taken as a parameter. We conclude that under the application of magnetic field the variable $y = T/T_M \propto T/(\mu_B(B - B_{c0})$ remains the same and the normalized effective mass is again governed by Eq. (6). Equation (6) is also valid when B is a variable and T is a fixed parameter, and $y = \mu_B(B - B_{c0})/T_M$ can be again considered as an effective normalized temperature. We note that the obtained results coincide with numerical calculations [8, 9, 10].

The above considerations of the effective mass dependence on temperature and magnetic field permit to construct the schematic phase diagram of the substance under consideration. This diagram is depicted in Fig. 1. We show two LFL regions separated by NFL one. The left LFL region may also contain magnetic long-range order or even superconductivity. The phase boundary in this region is the transition temperature of the corresponding phase transitions which are incited to taking place by FCQPT [8, 9, 12]. The right LFL region corresponds to that induced by magnetic field.

Figure 3 displays magnetic field dependence of normalized (by the values of function and its argument in the inflection point, see below) muon spin-lattice relaxation rate $1/T_1^\mu$ in $YbCu_{5-x}Au_x$ (x=0.6) [13] along with our theoretical B-dependence. To obtain the latter theoretical curve we (for fixed temperature and in magnetic field B) solve the Landau integral equation for quasiparticle energy spectrum $\varepsilon(\mathbf{k})$ (see Refs. [8, 9] for details) with special form of Landau interaction amplitude. Choice of the amplitude is dictated by the fact that the system has to be in the FCQPT point, which means that first three k-derivatives of the spectrum $\varepsilon(\mathbf{k})$ should equal zero. Since first derivative is proportional to the reciprocal quasiparticle effective mass $1/M^*$, its zero (where $1/M^* = 0$ and the effective mass diverges) just signifies FCQPT, see, e.g. Refs. [8, 9] for details. Zeros of two subsequent derivatives mean that the spectrum has an inflection point at Fermi momentum p_F so that the lowest term of its Taylor expansion is proportional to $(p - p_F)^3$ [10]. After solution of the integral equation, the obtained spectrum had been used to calculate an entropy $S(B, T = const)$, which, in turn, had been recalculated to the effective mass by virtue of well-known LFL relation $M^*(B, T) = S(B, T)/T$. We note that our calculations confirm the validity of Eq. (6). The final step was to use relation (3) to calculate the reciprocal relaxation time.

The normalization procedure deserves a remark here. Namely, since the magnetic field dependence (both theoretical and experimental) of $1/T_1^\mu$ does not have "peculiar points" like extrema, the normalization have been performed in the inflection point shown by the arrow in the inset to Fig. 1. To determine the inflection point precisely, we first differentiate $1/T_1^\mu$ over B, find the maximum of derivative and normalize the values of the function and the argument by their values in the inflection point. It is seen that such procedure immediately reveals the universal magnetic field behavior of the normalized reciprocal relaxation time $1/T_{1N}^\mu$, showing its proportionality to the effective mass square. We emphasize here

that the entire field (and temperature) dependence of $1/T_1^\mu$ is completely determined by corresponding dependence of the effective mass. The fact that M^* becomes field, temperature and other external parameters dependent is a key consequence of the FCQPT theory.

Consider now a longitudinal magnetoresistance (LMR) $\rho(B,T) = \rho_0 + AT^2$ as a function of B at fixed T. In that case, the classical contribution to LMR due to orbital motion of carriers induced by the Lorentz force is small, while the Kadowaki-Woods relation $K = A/\gamma_0^2 \propto A/\chi^2 = const$ [15] allows us to employ M^* to calculate $A \equiv A(B)$ [16]. As a result, $\rho(B,T) - \rho_0 \propto (M^*)^2$. Inset to Fig. 3 reports the normalized magnetoresistance

$$R_N^\rho(y) = \frac{\rho(y) - \rho_0}{\rho_{inf}} \propto \frac{1}{T_{1N}^\mu} \propto (M_N^*(y))^2 \tag{9}$$

vs normalized magnetic field $y = B/B_{inf}$ at different temperatures, shown in the legend. Here ρ_{inf} and B_{inf} are LMR and magnetic field taken at the inflection point shown in the inset to Fig. 1 by the arrow. The transition region where LMR starts to decrease is shown in the inset by the hatched area and takes place when the system moves along the horizontal dash-dot arrow. We note that the same normalized effective mass has been used to calculate both $1/T_{1N}^\mu$ and the normalized LMR. Thus, Eq. (9) determines the close relationship between the quite different dynamic properties, showing the validity of the extended Landau paradigm.

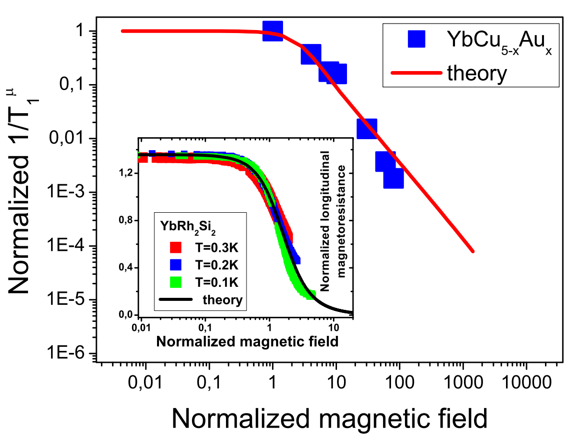

FIG. 3: Magnetic field dependence of normalized (in the inflection point, see text for details) muon spin-lattice relaxation rate $1/T_1^\mu$ in $YbCu_{4.4}Au_{0.6}$ [13] along with our theoretical B-dependence of the square of a quasiparticle effective mass (6). Inset shows the normalized magnetoresistance $R_N^\rho(y)$ versus normalized magnetic field. $R_N^\rho(y)$ was extracted from LMR of $YbRh_2Si_2$ at different temperatures [17] listed in the legend. The solid line represents our calculations.

Both theoretical and experimental curves have been normalized by their inflection points, which also reveals the universal behavior - the curves at different temperatures merge into a single one in terms of scaled variable y. Figure 3 shows clearly that both normalized magnetoresistance R_N^ρ and reciprocal spin-lattice relaxation time well obeys the scaling behavior given by Eq. (9). This fact obtained directly from the experimental findings is a vivid evidence that both above quantities behavior is predominantly governed by field and temperature dependence of the effective mass $M^*(B,T)$.

In summary, our theoretical study of $1/T_1T$ and LMR in two different HF compounds shows that their characteristic behavior is due to the dependence of the quasiparticle effective mass on magnetic field, temperature and other external parameters. Our results are in good agreement with experimental facts and allow us to confirm the validity of the extended Landau paradigm. This paradigm, in turn, permits us to explain for the first time the magnetic field behavior of both $1/T_1T$ and LMR.

This work was supported in part by the grants: RFBR No. 09-02-00056, DOE and NSF No. DMR-0705328, and Opole University Intramural Grant (Badania Statutowe).

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